Tunnelling times of electrons

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Abstract

We consider a fully relativistic method for the calculation of tunnelling times based on the streamlines of the conserved probability flux. This method is similar to that proposed by Leavens [1, 2] but is not based on the Bohmian interpretation. The method is applied to single and two-particle tunnelling in Dirac theory.

1 Introduction

There have been many attempts to define a physical time for quantum mechanical tunnelling processes, since the question was raised by MacColl [3] in 1932. Orthodox quantum theory is unable to give a definite answer to this problem since time is not an observable (in the sense of being represented by an Hermitian operator) in the conventional formulation. There is renewed interest in this problem due, in part, to recent experimental advances which have observed directly the times involved in optical photon and microwave analogues of quantum tunnelling [4]–[9]. Comprehensive reviews of the various approaches to the problem have been given by Hauge & Støvneng [10] and Landauer & Martin [11]. Of these approaches, only the dwell time introduced by Smith [12] is widely accepted. This is a statement of the average time spent in a given region of space averaged over all scattering channels. Other authors have proposed definitions of the duration of the tunnelling process which differentiate between those particles that are ultimately transmitted by the barrier and those that are reflected. These include (classical) phase times, the Larmor times suggested by Baz' [13], the Büttiker & Landauer [14] time and the complex time of Sokolovski & Baskin [15]. The Larmor, the Büttiker & Landauer and the Sokolovski & Baskin times are all attempts to define the length of time

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a transmitted particle spends inside the barrier region. These definitions may be extended to consider incident states that are not monochromatic [16, 17, 18] and to generate distributions for the times involved [19, 20]. The methods which we group under the title of phase times involve following the peaks of incident and transmitted wavepackets through the tunnelling event. This may be used to define a barrier interaction time, although the validity of this time is highly questionable [14].

Leavens and coworkers [1, 2, 21, 22] and others [23, 24, 25] have applied the Bohmian interpretation of non-relativistic quantum mechanics to the problem of one-dimensional tunnelling. In this approach (classical) particles follow the integral curves (streamlines) of the conserved probability current under the action of a new 'quantum force'. In this interpretation, the distribution of tunnelling times is obtained from the ensemble of times that the particles spend inside the barrier. In this paper we consider a similar, but fully relativistic method, for predicting times associated with the tunnelling process. This approach also uses the spacetime streamlines of a conserved current, but we do not require that these be identified with the trajectories of (classical) particles. Instead we require only that the flow of the probability density reflects the temporal aspects of the tunnelling process. By construction, the predictions of this approach will agree with orthodox quantum mechanics in those situations where orthodoxy is able to predict. However, streamline-based techniques go on to give unambiguous predictions even in situations where standard quantum theory cannot. The barrier traversal time is one such example.

This paper is concerned with electron tunnelling only, although we expect streamline methods to be of value in other situations as well (see, for example, Prosser [26, 27] for non-covariant applications to the classical electromagnetic field, and Holland [24] for other applications). In particular, an extension of streamline techniques to quantum-optical problems should aid the discussion of the recent two-photon experiments of Steinberg *et al.* [4, 5, 6]. Our single-particle calculations are based in first-quantised Dirac theory, thus ensuring that the streamlines always lie inside the forward light cone [28], contrary to predictions derived from the Schrödinger equation [10]. It follows that traversal times are always bounded above by the light travel time. Furthermore, we are assured that any spin-dependent effects are properly accounted for. We shall see that for a low energy electron, incident normally in a helicity eigenstate, our predictions agree with the earlier work of Leavens. We also discuss how these predictions are modified when the spin is varied. Finally, we consider the streamlines for a pair of normally incident electrons. In this case, the Pauli principle must be considered, and this has a significant effect on the distribution of interaction times.

2 The method

The approach we employ to predict the physical times associated with tunnelling processes between free asymptotic states is as follows. We require that the wavefunction determines a conserved probability flux field $J^\mu(\boldsymbol{x}, t)$, with the time component J^0 interpreted as the probability density (in the laboratory frame) for locating the particle at $(\boldsymbol{x}, t)$. The streamlines of this flux field never cross in spacetime, since they are the integral curves of a conserved vector field. Furthermore, the time component J^0 flows along these streamlines without 'leaking' between them. For this reason, the streamlines are a very useful tool for studying the flow of the particle's position probability density. The incident particle is represented by a wavepacket, whose future development is governed by fully deterministic equations. The streamlines of the flux field can then be followed through spacetime from the initial packet. Each streamline is assigned a statistical weight, given by the value of J^0 at its starting point in the initial wavepacket, and a parameter representing the time we wish to determine. For example, if we seek the length of time spent by a tunnelling particle in the barrier region, the parameter assigned to a given streamline is simply the amount of time τ the streamline spends in that region. The statistical ensemble of streamlines may then be used to calculate probability distributions for the time of interest. We believe it is natural to expect this distribution to be realised experimentally, given the standard interpretation of the conserved flux field. We have given a preliminary report of the application of this method to electron tunnelling elsewhere [28, 29]. When applied to the Schrödinger equation, the results of this approach coincide with those of Leavens *et al.* [1, 2, 21]. However, since this approach does not require that the streamlines determine the trajectories of (classical) particles, it is of wider applicability. In particular, we expect that quantum optical problems may be addressed without the need for the problematic concept of the photon trajectory [30, 31].

Consider a typical experiment where a particle, which we represent by a spatially localised wavepacket, is prepared at some time t_0, well outside the barrier region. Subsequently the particle interacts with the barrier and may be scattered into one of several scattering channels. We wish to calculate the distribution $P_B(t_B)$ for the length of time t_B the particle interacts with the barrier, given that it appears in some specified channel after the scattering event is completed. Let each streamline be identified by its starting position $\boldsymbol{x}_0$ in the initial packet at time t_0. We consider that region of $\boldsymbol{x}_0$ space formed from all streamlines that enter the scattering channel of interest. Each such streamline spends a time τ inside the barrier region. If we assume that $\tau(\boldsymbol{x}_0)$ is a differentiable function of $\boldsymbol{x}_0$ over the interior of this region, then the distribution $P_B(t_B)$ is given by

$$P_B(t_B) \propto \int \frac{J^0}{|\boldsymbol{\nabla}\tau|} dS, \tag{2.1}$$

where the integral is taken over the surface $\tau(\boldsymbol{x}_0) = t_B$, and dS is the magnitude of the surface element.

Now consider a one-dimensional experiment where the barrier is normal to the z direction, and the particle is localised only along this direction. The particle's motion is entirely along the z-axis. In this configuration the particle is either reflected or transmitted by the barrier. The surface in $\boldsymbol{x}_0$ space which divides these channels is now a plane parallel to the barrier. If we wish to calculate the distribution $P_A(t_A)$ of arrival times t_A at a planar screen located at z_A on the far side of the barrier, we would label each streamline with its arrival time at the plane z_A. This labelling is potentially ambiguous in the presence of back-flow, where a streamline may intersect the plane z_A more than once [22]. However, it appears that this may be avoided if the screen is sufficiently far from the barrier [32]. We will assume that this is the case. Evaluating the integral (2.1) we find that

$$P_A(t_A) \propto J^0(t_0, z_0)|dz_0/dt_A|, \tag{2.2}$$

where (t_A, z_A) and (t_0, z_0) are connected by a streamline. But the conservation law $\partial J^\mu/\partial x^\mu = 0$ implies that

$$|J^3(t_A, z_A)dt_A| = J^0(t_0, z_0)|dz_0|. \tag{2.3}$$

It follows that, in the absence of back-flow ($J^3 > 0$),

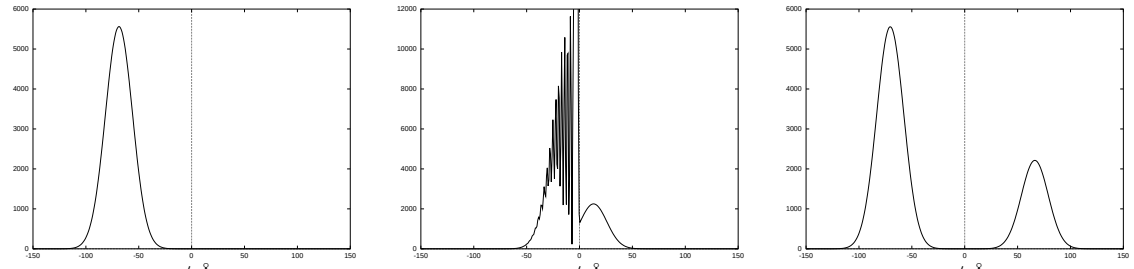
$$P_A(t_A) \propto J^3(t_A, z_A), \tag{2.4}$$

which states that, in one dimension, the distribution of arrival times at a point is proportional to the spatial current evaluated at that point. Expressions such as (2.4) were criticised by Allcock [33, 34, 35] who claimed that it was impossible to give an operational procedure which would yield an 'ideal' arrival time measurement (ideal in the sense that the result is independent of the details of the measuring device), and hence that the concept of arrival times has no place in standard quantum theory. Muga *et al.* [32] have shown that this claim is incorrect. By demonstrating the existence of the ideal absorber, they were able to show that the mean time for absorption of the probability density can be made arbitrarily close to the mean of the distribution (2.4).

Returning to barrier traversal times, it is simple to show [21] that the average time t_D a streamline is inside the barrier is given by

$$t_D = \int_{t_0}^{\infty}\!\!\int dt\, d^3\boldsymbol{x}\, J^0(\boldsymbol{x}, t), \tag{2.5}$$

where the spatial integration is taken over the region containing the barrier. This is equivalent to the expression for the dwell time derived in [15] from a path integral analysis.



(1a) (1b) (1c)

Figure 1: *The time component of the probability flux as a function of position at* $t = -5\,\mathrm{fs}$ *(Figure 1a),* $t = +1\,\mathrm{fs}$ *(Figure 1b) and* $t = +5\,\mathrm{fs}$ *(Figure 1c). In all plots the vertical scale to the right of the barrier is multiplied by* 10^4 *to enhance the features of the small transmitted packet. The peak of the transmitted wave is slightly advanced from where the peak of the wavefunction would have been in the absence of the barrier.*

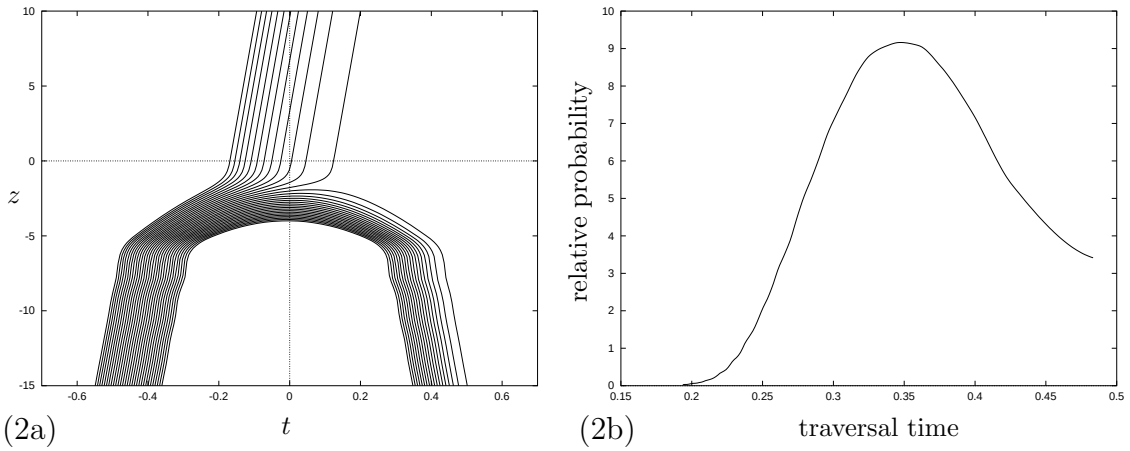
3 One-dimensional electron tunnelling

We shall consider an electron, moving along the z-axis, incident from the left on a potential barrier of height E_b. The barrier extends infinitely in the plane normal to the z direction. To avoid the problems associated with pair production at the barrier we shall suppose that $2mc^2 > E_b - E_{kin}$ where m is the mass of the electron and E_{kin} the electron's kinetic energy outside the barrier region. We represent the electron by a one-dimensional wavepacket localised along the direction of motion. Furthermore, each plane wave component of the wavepacket will be taken to be a helicity eigenstate. With these conditions we eliminate transverse currents in the barrier region, and so have a genuinely one-dimensional problem. In the case of Dirac theory, the required conserved flux 4-vector is given by

$$J^\mu = \overline{\psi}(\boldsymbol{x}, t)\gamma^\mu\psi(\boldsymbol{x}, t). \tag{3.1}$$

We calculate the streamlines of the Dirac flux field $J^\mu(\boldsymbol{x}, t)$ and follow these up to the barrier, and, if not reflected, beyond the barrier. Those streamlines that cross the barrier then determine distributions for the time spent in the barrier region.

The interaction of a Gaussian wavepacket with a potential barrier is shown in Figure 1. The wavepacket has central energy $E_{kin} = 5\,\mathrm{eV}$ and width $\Delta k = 0.04\,\mathrm{Å}^{-1}$ in momentum space. The barrier has height $E_b = 10\,\mathrm{eV}$ and extends from $z = -5\,\mathrm{Å}$ to $z = 0\,\mathrm{Å}$. These parameters are chosen to allow direct comparison with the work of Leavens and Aers [21]. The corresponding packet in position space is shown in Figure 1, which plots the time component J^0 of the Dirac probability flux as a function of position at $t = -5\,\mathrm{fs}$, $t = +1\,\mathrm{fs}$ and $t = +5\,\mathrm{fs}$. At $t = 5\,\mathrm{fs}$ we see that the peak of the transmitted packet lies at $z = 67\,\mathrm{Å}$, while the peak of the initial



(2a) (2b)

Figure 2: *Particle streamlines and time spent in the barrier. Figure 2a shows the streamlines for the front of the wavepacket, indicating that only the streamlines from the front of the packet cross the barrier. Each streamline slows down as it passes through the barrier. Figure 2b is a histogram of time the streamlines spend in the barrier. Distance is measured in* Å *and time in* 10^{-14}s.

packet would have been at $z = 65\,\text{Å}$ had the barrier not been present. The size of this advance, divided by the group velocity $4.4 \times 10^{-3}c$, gives a time advance of $\sim 0.2\,\text{fs}$, so that the transmitted wave appears to take $0.2\,\text{fs}$ *less* time to pass through the barrier than to pass through an equal path-length of free space. This result is often interpreted as meaning that the electron, on average, spends less time in the barrier region when the barrier is present, than if it were absent. But this is a misinterpretation of the result. The only prediction that standard quantum theory allows us to make is that if we had some device that allowed production of electrons at a given time to the left of the barrier, and we timed the arrival of the transmitted electrons, the peak of the resultant distribution of arrival times would be shifted to earlier times by $0.2\,\text{fs}$ when the barrier was inserted.

A sample set of streamlines from the initial wavepacket is shown in Figure 2, along with a histogram of the time that the transmitted streamlines spend inside the barrier. The histogram is calculated from (2.1) with $d\tau/dz_0$ evaluated numerically from the streamline data. It is significant that a continuously distributed set of initial input conditions (the positions within the initial wavefunction from which the streamlines start) gives rise to a set of disjoint outcomes (whether or not a streamline passes through the barrier). In this case, deterministic evolution of the wavefunction alone is able to explain the discrete results expected in a quantum measurement. This is of fundamental significance to the interpretation of quantum mechanics. Some consequences of this view — though starting from the Bohmian interpretation of non-relativistic quantum mechanics — have been explored by Dewdney *et al.* in other areas of quantum measurement [25].

Figure 2 shows that only the streamlines starting near the front of the initial wavepacket pass through the barrier. They therefore have a 'head start' as regards their arrival at some chosen point on the far side of the barrier. Over the front part of the barrier, however, the transmitted streamlines slow down to speeds very much less than the group velocity in free space; this is responsible for the comparatively long time the transmitted streamlines spend in the barrier ($\sim 3.5\,\mathrm{fs}$), whilst both competing effects contribute to the arrival times at the chosen finishing point. We can compare this value for the time spent under the barrier with the predictions of previous authors. For example, the Büttiker & Landauer [14] time (interaction time with an oscillating barrier) gives a value of 0.38 fs for our electron simulation. This is an order of magnitude smaller than the average time the streamlines spend under the barrier, and, for our chosen parameters, it is equal to the time taken by a free electron to traverse the equivalent distance in free space. The results presented in Figure 2 are in excellent quantitative agreement with those of Leavens, for the same parameters.

Figure 1 demonstrates that the effect of selecting streamlines from the front of the packet outweighs their subsequent slowing down, shifting the distribution of arrival times to earlier times by $\sim 0.2\,\mathrm{fs}$. Similar shifts in arrival times have been observed in the photon experiments performed by Steinberg *et al.* [4, 6]. The preferential selection of the streamlines from the front of the packet is a graphic illustration of the pulse reshaping that underlies this phenomenon.

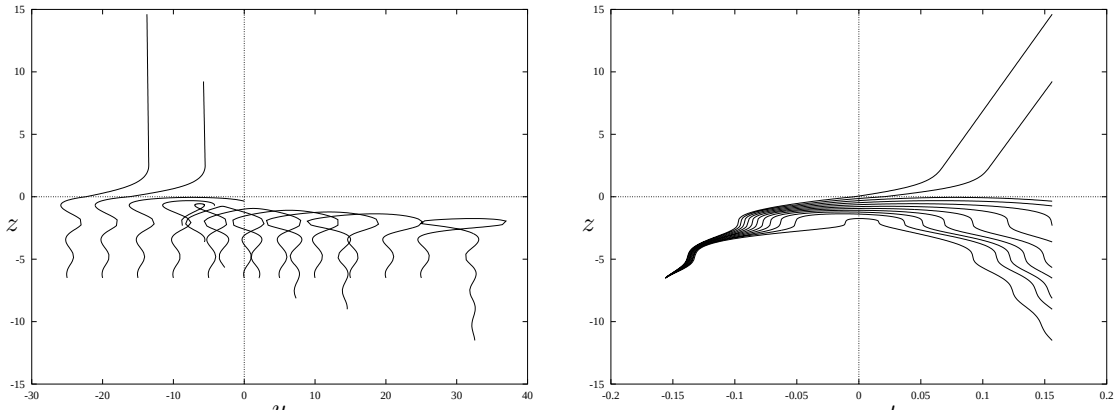
4 Spin dependent effects

The analysis of the previous section was simplified by aligning the spin along the normal to the barrier, and considering a packet composed of plane waves with momenta also along this direction. For other spin orientations, or for oblique incidence, we must consider wavepackets that localise the incident particle in two or three-dimensions. In this section we consider briefly the spin dependence of arrival times and traversal times for normal incidence.

We first consider the distribution of arrival times as measured by a detector which responds to particles arriving over some region S of the plane $z = z_A$. The analogue of (2.4) for the arrival time distribution is

$$P_A(t_A) \propto \int dx\, dy\, J^3(\boldsymbol{x}, t_A), \tag{4.1}$$

where the integral is taken over the surface S, and we have assumed that $J^3 > 0$ over this surface. We consider the tunnelling of a particle of mass m, and kinetic energy E_{kin}, through a barrier of height E_b. The barrier has width d and lies in the $x - y$ plane. It is straightforward to show that the transmission coefficient T is spin



(2a) (2b)

Figure 3: *A two-dimensional simulation. Figure 3a shows streamlines in the y-z plane, which are all started off at the same time. Clearly the left-hand side of the packet, which 'spins' into the barrier, is preferentially transmitted. Figure 3b shows the same streamlines in the z-t plane. Distance is measured in* Å *and time in* 10^{-14} s.

independent. In units where $\hbar = c = 1$, we have that

$$T = \frac{e^{-ip_z d} \kappa p_z}{\kappa p_z \cosh(\kappa d) - i(p_z^2 - EE_b)\sinh(\kappa d)}, \tag{4.2}$$

where p_z is the component of the incident momentum along the normal to the barrier, E is the total energy ($E = E_{kin} + m$), and κ is given by

$$\kappa^2 = -p_z^2 - E_b(E_b - 2E) \qquad |E - E_b| < \sqrt{E^2 - p_x^2}. \tag{4.3}$$

The spin independence of the transmission coefficient ensures that there is no change of spin polarisation on transmission [36]. The stationary phase approximation suggests that the position of the peak of the transmitted packet is the same for incident packets that differ only in spin polarisation. However, the arrival time distribution, given by (4.1) will be weakly spin dependent because the current $J^\mu(\boldsymbol{x}, t)$ in a wavepacket contains oscillating components which are spin dependent [37].

To consider the effect of spin polarisation on the traversal time distribution, we consider a specific two-dimensional example. We construct a wavepacket where the average momentum is along the normal to the barrier, with all momentum components lying in a plane. Such a packet represents a normally incident particle localised in two-dimensions, which we take to be the y-z plane. Provided that we take the incident spin polarisation to be along the normal to this plane (the x direction), then the spatial current is confined to the plane of incidence.

In Figure 3 we show the streamlines for such an arrangement, where the incident Gaussian wavepacket has a width $\Delta k = 0.04\,\text{Å}^{-1}$ and kinetic energy 5 eV. The

barrier is at $z = 0$ and has width 2.5 Å and height 10 eV. The incident packet has spin polarisation along the $-x$ direction. Transverse currents are clearly displayed in Figure 3. The motion near the barrier is highly complex; the appearance of closed loops of current are suggestive of the formation of vortices. Similar effects have been reported by Hirschfelder *et al.* [38] in the context of Schrödinger theory. It is clear from the Figure that the packet no longer divides (in $\boldsymbol{x}_0$ space) in the plane of the barrier. That part of the packet which 'spins' into the barrier is preferentially transmitted. Numerical investigations with three-dimensional wavepackets of arbitrary spin polarisation suggest that the packet divides over a spin dependent surface, but qualitatively it is still the front part of the packet which 'spins' into the barrier which is more strongly transmitted. It seems likely that this will be the most significant effect controlling the spin dependence of the traversal times. The streamline plots show that the component of the streamline velocity normal to the barrier is lower inside the barrier than in the freely propagating packet. This effect offsets the fact that it is the front of the wavepacket that crosses the barrier, in determining the distribution of arrival times.

In principle, we could calculate the traversal time distribution from (2.1), with the streamline traversal time $\tau(\boldsymbol{x}_0)$ calculated directly from the streamlines. However, the need to identify the level surfaces of $\tau(\boldsymbol{x}_0)$ suggests that this distribution is far from trivial to evaluate. The mean traversal time should be easier to calculate, since it is given simply by an integral of τ over the region of $\boldsymbol{x}_0$, from which transmitted streamlines start. Fortunately, many of the qualitative features of these distributions may be seen from inspection of the streamline plots alone.

5 Two particle tunnelling

The method employed by Leavens *et al.* [1, 2] is easily extended to more than one particle, since single-particle Schrödinger theory generalises to a multiparticle theory in a straightforward manner. For a two particle theory the wavefunction is of the form $\psi(\boldsymbol{r}, \boldsymbol{s}, t)$, where $\boldsymbol{r}$ and $\boldsymbol{s}$ are the spatial positions of the two particles. The dependence of ψ on both $\boldsymbol{r}$ and $\boldsymbol{s}$ introduces spatial correlations between the two particles. The straightforward generalisation of this method to the Dirac equation results in a theory which is not manifestly Lorentz covariant [39]. In [29] we outlined a manifestly covariant wavefunction-based approach to multiparticle relativistic quantum theory. This employed a wavefunction $\psi(r, s)$, where r and s are the spacetime positions of the two particles. Such a wavefunction clearly encodes correlations between the spacetime motions of the two particles. In this section we sketch the outline of an extension of the single-particle streamline method to the multiparticle domain. We shall assume that the particles interact with the external field, but not directly with each other.

We shall restrict attention to states in the direct product space of two single-

particle spinors. Such states may always be written as

$$\psi = \sum_j \phi_j \otimes \chi_j, \tag{5.1}$$

where ϕ_j and χ_j are single-particle Dirac spinors. The adjoint $\bar{\psi}$ for such states is defined by

$$\bar{\psi} \equiv \sum_j \bar{\phi}_j \otimes \bar{\chi}_j, \tag{5.2}$$

where the single-particle adjoint takes its usual meaning.

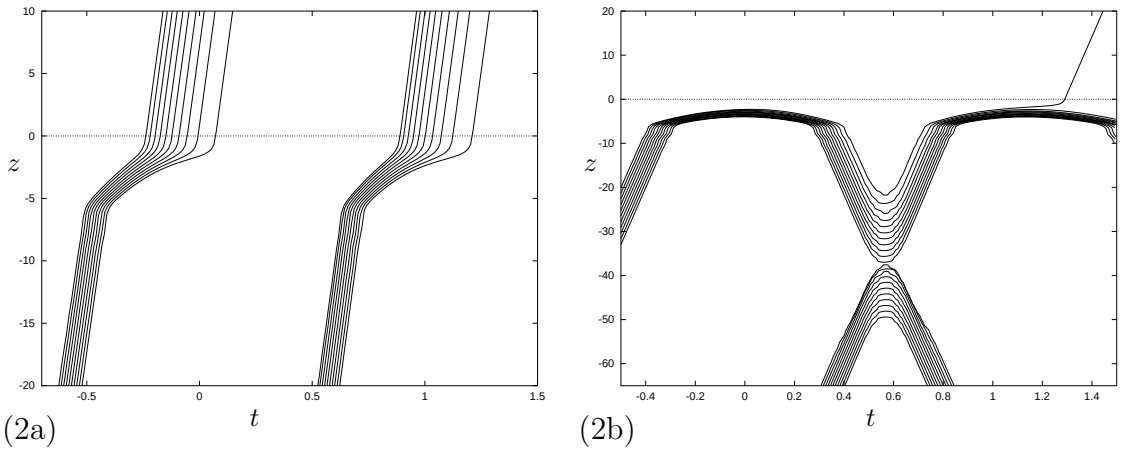
In [29], we proposed a Lorentz-covariant equation of motion for ψ which employed anti-commuting operators from the different spaces. The associated conserved current (in the two-particle configuration space) J has components J_1^μ associated with particle 1 and J_2^μ with particle 2, where

$$J_1^\mu = \bar{\psi}\gamma^\mu \otimes I\psi \qquad J_2^\mu = \bar{\psi}I \otimes \gamma^\mu\psi, \tag{5.3}$$

where I is the identity operator. The streamlines of the current in configuration space may be projected into pairs of correlated streamlines $r^\mu(\lambda)$ and $s^\mu(\lambda)$ in spacetime. The parameter λ encodes correlations between the spacetime behaviour of the two individual streamlines. Similar ideas were discussed in the recent paper by Berndl *et al.* [40].

By supposing that these streamlines represent the correlated trajectories of classical particles, we would arrive at the relativistic analogue of the multiparticle Bohmian interpretation. An immediate problem with this interpretation is that the projected streamlines are not necessarily timelike curves. For this to happen requires the presence of negative energy states, and it is possible that this means that the method may offer a 'realistic' description of pair production and annihilation processes. These ideas will be expanded elsewhere.

For a wavefunction of the form $\psi = \phi(r) \otimes \chi(s)$, a streamline in configuration space projects to a pair of spacetime streamlines which coincide with integral curves of the one-particle currents formed from ϕ and χ. This will be the case for distinguishable particles. We suggest that the projected streamlines may still provide useful information about the kinematics of the tunnelling process, even for non-factored states. In the tunnelling situation described here, the projected currents are always timelike since pair production is not important. In such cases, there is no possibility of superluminal transmission of a streamline. To extract quantitative information we require a probabilistic measure (weight function) on the space of correlated streamlines. Tunnelling time distributions would then be obtained in the same manner as for the single-particle case. The details of this weighting procedure will be given elsewhere. Here, we just wish to explore some of the qualitative features of the distributions obtained with this method when the particles are indistinguishable.



(2a) (2b)

Figure 4: *Correlated streamlines for a pair of identical, non-interacting fermions incident on a potential barrier. The streamlines start off from equal time points* $r^0 = s^0$*, with an initial separation of* 150 Å *along the* z *direction. Figure 4a shows streamlines from the front of the two wavepackets. Figure 4b shows streamlines from the back of the two packets. In the latter case, exchange terms are clearly significant. Distance is measured in* Å *and time in* 10^{-14}s.

In Figure 4 we consider some of the correlated streamlines for a pair of non-interacting identical electrons incident normally on a potential barrier. The barrier has height 10 eV and width 5 Å. Since the electrons are identical and non-interacting, we may assume a wavefunction of the form

$$\psi(r,s) = \frac{1}{\sqrt{2}}(\phi(r) \otimes \chi(s) - \chi(r) \otimes \phi(s)), \tag{5.4}$$

where ϕ and χ are solutions to the single-particle problem. In Figure 4, ϕ is the solution for a normally incident, one-dimensional Gaussian wavepacket, with $E_{kin} =$ 5 eV and $\Delta k = 0.04\,\text{Å}^{-1}$, with spin aligned along the incident direction. The spinor χ employs the same initial wavepacket as ϕ but is displaced by -150 Å along the normal to the barrier (the z direction). The streamlines shown in Figure 4 are started off from points in configuration space with equal time coordinates in the laboratory frame, and with a constant separation of 150 Å. For streamlines starting from the front of each packet, the correlated streamlines are similar to those obtained for distinguishable particles, where antisymmetrisation of the wavefunction is not required. For streamlines started off from behind the single-particle bifurcation points in each wavepacket, there is an initial reflection from the barrier. After reflection the streamline approaches a point in configuration space with $r = s$, from which it is repelled [29]. For an antisymmetrised wavefunction, the currents J_1^μ and J_2^μ are equal at such points. From Figure 4, it is clear that the current components J_1^3 and J_2^3 have opposite sign as this point is approached, so these components must

vanish there. It follows that the pair of correlated spacetime streamlines repel each other at this point. After the repulsion one of the pair is incident again on the barrier, and there is a small probability of further transmission.

The plots in Figure 4 should be compared to those given by Leavens *et al.* [21] for a single particle, represented by a wavefunction which is initially a coherent superposition of two widely separated Gaussian components. In the latter case all of the transmitted streamlines start from the first component, since the single particle streamlines cannot cross. Landauer & Martin [11] cited this example in a review of some of the unexpected features resulting from the Bohmian view. Figure 4 demonstrates that if we reinterpret this situation as describing two incident particles, then streamlines from the second component are able to cross the barrier.

The generalisation of the single-particle method for calculating tunnelling time distributions, discussed above, involves weighting correlated streamlines with an appropriate weight function, and then adding up the weights for those streamlines that 'describe' the dynamical situation of interest. For example, one could ask for a distribution of arrival times for the transmitted particle, given that one particle is transmitted and one reflected. Such a distribution would include a contribution from the pair of streamlines in Figure 4 where one member of the pair is transmitted on its second attempt. In this manner, the Pauli exclusion principle has a significant effect on the distributions of tunnelling times.

6 Conclusions

We have presented a streamline-based approach for the calculation of tunnelling time distributions. The results of this method may be regarded as the relativistic generalisations of those of Leavens [1, 2]. However, we have chosen not to interpret the streamlines as the world-lines of (classical) particles, as in the Bohmian approach employed by Leavens. Our use of the streamlines is restricted to following the flow of probability density through space. The method is based on the assumption that the dynamics of the probability density flow directly describe the temporal aspects of the tunnelling process. The arrival time distribution calculated on the basis of this assumption is in agreement with the mean arrival time predicted by Muga *et al.* [32], who employed an operational model of the detection process. Their work refuted earlier claims [33, 34, 35] that the concept of arrival times has no basis in standard quantum theory.

The status of the predictions for the traversal times is less clear. Any 'measurement' of the elapsed time between entering and leaving the barrier will disturb the system. This makes the concept of a traversal time problematic in the conventional interpretation, since the 'observable' cannot be related to the results of ideal operational model experiments. This is not the case in the Bohmian interpretation, where the underlying theory allows intrinsic properties to be defined in terms of the

particle trajectories alone. We share the view of Landauer & Martin[11] that the concept of a traversal time, while being useful, is not indispensable. Different processes involving tunnelling may well have different relevant timescales, all of which may be used to define a traversal time. In this letter we have taken the viewpoint that the flow of probability density provides a natural *definition* of a traversal time. It remains to be seen for what tunnelling processes this timescale will be relevant.

A consideration of general spin orientations suggested a picture where the part of the wavepacket which 'spins' into the barrier is preferentially transmitted. Tunnelling time distributions may still be calculated in these more general cases, although the complexity of the numerical work is greatly increased.

Finally we have considered the problem of two particle tunnelling, in a manifestly Lorentz covariant framework. We have outlined an extension of the relativistic streamline method to multiparticle situations. This involves a configuration space with multiple time coordinates, and may be regarded as the relativistic generalisation of the multiparticle Bohmian approach [24]. Our treatment of this problem is not yet complete, but we are confident that the current problems will be resolved in the future. Our future work in this area will concentrate on multiparticle methods and an extension of the streamline method to quantum-optical problems.

7 Acknowledgements

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