Physical Applications of Geometric Algebra

Sample Tripos Questions

1. Outline the role of the *vector derivative* in *four* of the following topics
i. The Cauchy-Riemann equations.
ii. Hamiltonian dynamics.
iii. The Maxwell equations.
iv. The Dirac Equation.
v. Gravitation.

2. A rotating frame $\{f_k(t)\}$ is related to the fixed frame $\{e_k\}$ by the rotor $R(t)$ via

$$f_k(t) = R(t)e_k\tilde{R}(t).$$

Explain why the rotor R satisfies the equation

$$\dot{R} = -\tfrac{1}{2}\Omega_S R = -\tfrac{1}{2}R\Omega_B$$

What is the significance of the bivector Ω_S? [5]

Outline the steps by which the angular momentum L of a rigid body is written in the form

$$L = R\mathcal{I}(\Omega_B)\tilde{R}$$

where $\mathcal{I}(B)$ is the inertia tensor

$$\mathcal{I}(B) = \int d^3x\, \rho x\wedge(x\cdot B)$$

and ρ is the density. [7]

A symmetric top has moments of inertia i_1, i_1, i_3 in the Ie_1, Ie_2, Ie_3 planes respectively. By writing

$$\Omega_S = \sum_k \omega_k I f_k$$

show that we can write [3]

$$L = i_1(\omega_1 I f_1 + \omega_2 I f_2) + i_3\omega_3 I f_3.$$

Hence show that for a symmetric top [2]

$$\Omega_S = \frac{1}{i_1}L + \frac{i_1 - i_3}{i_1}\omega_3 R I e_3 \tilde{R}$$

Given that L and ω_3 is constant, show that the rotor equation is solved by

$$R(t) = \exp(-\tfrac{1}{2}\Omega_l t)R(0)\exp(-\tfrac{1}{2}\Omega_r t).$$

and find expressions for Ω_l and Ω_r. [3]

3. Write an essay on the role of rotors in *relativistic* physics. Your essay should include the following topics: Lorentz transformations, rotors and bivectors; the acceleration bivector; the Lorentz force law; Dirac spinors; gauge theory and gravity.