Physical Applications of Geometric Algebra

Formula Sheet

Geometric Product

$$ab = a\cdot b + a\wedge b$$
$$ba = a\cdot b - a\wedge b$$
$$a\cdot A_r = \tfrac{1}{2}(aA_r - (-1)^r A_r a)$$
$$a\wedge A_r = \tfrac{1}{2}(aA_r + (-1)^r A_r a)$$

Multivectors

$$A = \langle A\rangle_0 + \langle A\rangle_1 + \cdots = \textstyle\sum_r \langle A\rangle_r$$
$$A_r B_s = \langle A_r B_s\rangle_{|r-s|} + \langle A_r B_s\rangle_{|r-s|+2} + \cdots + \langle A_r B_s\rangle_{r+s}$$
$$A_r\cdot B_s = \langle A_r B_s\rangle_{|r-s|}$$
$$A_r\cdot\lambda = 0 \quad \text{for scalar } \lambda$$
$$A_r\wedge B_s = \langle A_r B_s\rangle_{r+s}$$
$$\langle AB\rangle = \langle BA\rangle$$
$$A_r\cdot(B_s\cdot C_t) = (A_r\wedge B_s)\cdot C_t \quad r+s\le t \text{ and } r,s>0$$
$$A_r\cdot(B_s\cdot C_t) = (A_r\cdot B_s)\cdot C_t \quad r+t\le s$$
$$A\times(B\times C) + C\times(A\times B) + B\times(C\times A) = 0$$

Frequently Used. a, b, c, d vectors, B bivector.

$$a\cdot(b\wedge c) = a\cdot b\, c - a\cdot c\, b$$
$$(a\wedge b)\cdot(c\wedge d) = a\cdot d\, b\cdot c - a\cdot c\, b\cdot d$$
$$a\cdot(b\cdot B) = (a\wedge b)\cdot B$$
$$(a\wedge b)\times(c\wedge d) = b\cdot c\, a\wedge d - a\cdot c\, b\wedge d + a\cdot d\, b\wedge c - b\cdot d\, a\wedge c$$
$$(a\wedge b)\times B = (a\cdot B)\wedge b + a\wedge(b\cdot B)$$

Bivectors.

$$BC = B\cdot C + B\times C + B\wedge C$$
$$CB = B\cdot C - B\times C + B\wedge C$$
$$B\times A_r = \langle B\times A_r\rangle_r$$
$$B\times(MN) = B\times M\, N + M\, B\times N \quad \forall M, N$$

Pseudoscalar

$$A_r\cdot(B_sI) = A_r\wedge B_s\, I \qquad r+s \leq n$$
$$A_r\wedge(B_sI) = A_r\cdot B_s\, I \qquad r \leq s$$

Reflections and Rotations

$$A_r \mapsto (-1)^r n A_r n$$
$$A_r \mapsto R A_r \tilde{R}$$
$$R = \mathrm{e}^{-B/2} = kl\cdots mn$$
$$R\tilde{R} = \tilde{R}R = 1$$

Linear Algebra

$$\mathsf{f}(a\wedge b\wedge\cdots\wedge c) = \mathsf{f}(a)\wedge\mathsf{f}(b)\wedge\cdots\wedge\mathsf{f}(c)$$
$$\mathsf{f}(I) = \det(\mathsf{f})\, I$$
$$\bar{\mathsf{f}}(b) = e_k\, \mathsf{f}(e^k)\cdot b = \partial_a\, \mathsf{f}(a)\cdot b$$
$$\mathsf{f}(A_r)\cdot B_s = \mathsf{f}[A_r\cdot\bar{\mathsf{f}}(B_s)] \quad r \geq s$$
$$A_r\cdot\bar{\mathsf{f}}(B_s) = \bar{\mathsf{f}}[\mathsf{f}(A_r)\cdot B_s] \quad r \leq s$$
$$\mathsf{f}^{-1}(A) = \det(\mathsf{f})^{-1} I\bar{\mathsf{f}}(I^{-1}A)$$
$$\bar{\mathsf{f}}^{-1}(A) = \det(\mathsf{f})^{-1} I\mathsf{f}(I^{-1}A)$$

Frames and contractions

$$e^i\cdot e_j = \delta^i_j$$
$$e^k = (-1)^{k+1} e_1\wedge\ldots\wedge\check{e}_k\wedge\cdots\wedge e_n E_n^{-1}$$
$$E_n = e_1\wedge e_2\wedge\cdots\wedge e_n$$
$$e_k\, e^k\cdot A_r = \partial_a\, a\cdot A_r = rA_r$$
$$e_k\, e^k\wedge A_r = \partial_a\, a\wedge A_r = (n-r)A_r$$
$$e_k A_r e^k = \dot{\partial}_a A_r \dot{a} = (-1)^r(n-2r)A_r$$

Geometric Calculus

$$\nabla(AB) = \nabla AB + \dot{\nabla}A\dot{B}$$
$$\nabla\wedge\nabla = 0$$
$$\nabla(x\cdot a) = a$$
$$\nabla x^2 = 2x$$